Filter Medium Clogging During Cake Filtration

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Medium clogging occurs frequently in cake filtration processes (Granger et al., 1985). Notebaert et al. (1975) and Tiller et al. (1981) have proposed an exponential-type empirical equation for correlating the medium resistance (1/m) R_m data as follows

$$R_m = R_{m\infty} - (R_{m\infty} - R_{m0})\exp(-jw_c^*), \tag{1}$$

where R_{m0} and $R_{m\infty}$ are respectively the initial medium resistance (l/m) and the resistance at infinite time limit, j an empirical parameter (m^2/kg) and w_c^* the dry cake mass per unit filter area (kg/m^2). Leu (1981) and Tiller et al. (1981) have satisfactorily employed Eq. 1 for correlating their experimental data.

Since Eq. 1 is entirely empirical, it seems reasonable to attempt a theoretical justification for its validity. Furthermore, according to Eq. 1, the rate of increase of medium resistance with w_c^* (dR_m/dw_c^*) should always decrease monotonously as the cake grows, that is, that no inflection point appears on an R_m-w_c^* plot. However, some published data sets have revealed that a clear inflection point is observable (Leu, 1981; Leu and Tiller, 1983), thereby implying the necessity for revising the above correlation equation.

Birth-death stochastic analysis has been widely employed to describe the dynamic behavior in deep-bed filtration (Hsu and Fan, 1984; Fan et al., 1985a,b,c; Nasser et al., 1986). Since the blocking of the filter membrane, however, is usually of an irreversible nature, it can be modeled as a pure-birth process. This work focuses on two aspects. First, the irreversible medium clogging process was modeled as a pure-birth stochastic process, from which a more comprehensive medium clogging equation is derived and compared with the experimental data. Second, the physical interpretations of the parameters involved in the newly derived clogging equation (including the empirical parameter j in Eq. 1) are clearly addressed.

Analysis

Medium resistance

At the onset of a filtration test all the pores are clean and open, but as cake grows in the course of the test, some pores on the filter medium eventually become clogged (denoted as pores C), while others always remain open (denoted as pores F). The total pressure drop is the sum of the pressure across the filtration cake ($|\Delta P_c|$) and that across the filter medium (Pa) ($|\Delta P_m|$). The filter medium resistance is defined as

$$R_m = |\Delta P_m|/\mu q_l, \tag{2}$$

where μ and q_l are the filtrate viscosity (Pa·s) and the superficial velocity (m^3/m^2·s), respectively. During a test, the pressure drop ΔP_m can either remain constant or vary, however, the corresponding flow field through filter medium can assume having reached steady state. (Note: this is a good approximation since the pressure drop variation is usually not rapid in the filtration process.)

Assume that on the unit medium area, there exist n_0 identical pores C. For subsequent analysis, the only requirement for a pore C is that the filtrate flow rate through it is $k|\Delta P_m|$. k is the proportionality parameter (m^3/Pa·s). (For example, if the pores are long, cylindrical holes of diameter D (m) and length L, $L \gg D$, then $k = \pi D^4/128L\mu$. As long as the proportionality relation holds, other types of pores may also be applicable.) In terms of Eq. 1, the above-mentioned requirement is equivalent to a constant hydraulic resistance for each open pore C ($1/k\mu$). During filtration, some pores C may gradually become irreversibly clogged. As a result, the number of clogged pores C, n, increases as the cake grows.

Apart from the constraint for pores F that the hydraulic resistance be a constant $R_{m\infty}$, that is, the final medium resistance, no other specific restriction applies to these pores.

At any instant, the filtrate superficial velocity through the whole cake is contributed by pores C and F as follows

$$q_l = k(n_0 - n)|\Delta P_m| + \frac{|\Delta P_m|}{\mu R_{m\infty}}. \tag{3}$$

Together with the basic definition of Eq. 2, Eq. 3 can be expressed as follows

$$\frac{1}{R_m} = \mu k(n_0 - n) + \frac{1}{R_{m\infty}}. \tag{4}$$

At time zero, $n = 0$ (no clogging) and $R_m = R_{m0}$. Equations 2–4 can thereby be rearranged to give

$$\frac{R_m - R_{m0}}{R_{m\infty} - R_{m0}} = \frac{n}{n_0}\frac{1}{1+\lambda(1-n/n_0)}, \tag{5}$$

where $\lambda = \mu k n_0 R_{m\infty} = |\Delta P_m| k n_0/(|\Delta P_m|/\mu R'_{m\infty}$ is the ratio of superficial velocities between (clean) pores C and pores F. This parameter therefore accounts for how seriously the medium may become clogged at infinite time limit.

The remaining problem is to estimate the clogged pore number n/n_0, a task for the following stochastic analysis.

Pure birth process

The derivations discussed in this section correspond closely to the findings of Fan et al. (1985c). However, herein is the first attempt to apply their more general stochastic analysis to the medium clogging problem. In filtration literature, the dry cake weight per unit filter medium area (w_c^*) rather than the filtration time is usually employed to correlate the medium clogging process (Tiller et al., 1981; Leu and Tiller, 1983). The simple reason is that as the volume of particles deposited on the filtration cake surface increases, more fine particles can probably reach the medium and clog the open pores. During a filtration test, both the cake resistance and the medium resistance change continuously, thereby yielding a varying filtrate flow rate. As a result, relating the medium clogging to the filtration time becomes problematic. Therefore, we here employ w_c^* as an independent variable in the following analysis.

Pore clogging on the medium's surface is governed by probabilistic laws. The number of clogged pores per unit area at dimensionless dry cake mass w_c ($= w_c^*/w_{c,\mathrm{ref}}$, where $w_{c,\mathrm{ref}}$ is the reference dry cake weight, equal to 1 kg/m^2) is the random variable under investigation. Assume that the probability for an open pore C at w_c becoming clogged during the filtration period (w_c, $w_c + 1$) is a constant β. Denote the specific value of $N(w_c)$ as n, the conditional probability that one open pore C at w_c becomes clogged during (w_c, $w_c + \Delta w_c$) is thereby $\beta(n - n_0)\Delta w_c$. Apparently, the probability of all open pores at w_c remaining open at $w_c + \Delta w_c$ is $1 - \beta(n - n_0)\Delta w_c$. N is the random variable for number of clogged pores C.

Take the probability that exactly n pores are clogged at w_c as $P_n(w_c)$, where $n = 0, 1, \ldots, n_0$. The following equalities hold for n larger than zero (P_n is the probability for exactly n pores C are clogged).

$$P_n(w_c + \Delta w_c) = \beta[n_0 - (n-1)]\Delta w_c P_{n-1}(w_c) + [1 - \beta(n_0 - n)\Delta w_c]P_n(w_c), \tag{6}$$

or equal to zero

$$P_0(w_c + \Delta w_c) = (1 - \beta n_0 \Delta w_c)P_0. \tag{7}$$

Take $\Delta w_c \to 0$ limit, Eqs. 6 and 7 can be reduced to the following master equations

$$\frac{dP_n(w_c)}{dw_c} = \beta[n_0 - (n-1)]P_{n-1}(w_c) - \beta(n_0 - n)P_n(w_c), \tag{8}$$

for $n > 0$, or

$$\frac{dP_0(w_c)}{dw_c} = -\beta n_0 P_o(w_c). \tag{9}$$

The initial conditions are $P_n(0) = 0$ for $n > 0$, and $P_0(0) = 1$.

The solution for Eqs. 8 and 9 is found for $n > 0$ as (Chiang, 1980)

$$P_n(w_c) = (-1)^n \lambda_0 \ldots \lambda_{n-1} \sum_{i=0}^{n} \exp(-\lambda_i w_c) / \prod_{l=0, l\neq i}^{n} (\lambda_i - \lambda_l), \tag{10}$$

for $n > 0$, where $\lambda_i = \beta(n_0 - i)$. i is a dummy variable. For $n = 0$, we have

$$P_0(w_c) = \exp(-\beta n_0 w_c). \tag{11}$$

Clearly the probability of pore clogging increases with the increase in dry cake mass.

The mean for $N(w_c)$ had been obtained by Fan et al. (1985c) as follows ($\langle n \rangle$ is the mean value of n):

$$\langle n \rangle = n_0[1 - \exp(-\beta w_c)]. \tag{12}$$

The substitution of Eq. 12 into Eq. 5 leads to the following expression for the medium resistance (Φ is the resistance ratio defined)

$$\Phi = \frac{R_m - R_{m0}}{R_{m\infty} - R_{m0}} = \frac{1 - \exp(-\beta w_c)}{1 + \lambda \exp(-\beta w_c)}. \tag{13}$$

Two parameters exist in Eq. 13: λ and β, with the former accounting for the relative importance of superficial velocities through (clean) pores C and pores F, and the latter, the ease of clogging. Some calculation results of Eq. 13 are shown in Figure 1.

If $\lambda \gg 1$, or the medium clogging is serious, Eq. 13 can be simplified as

$$\Phi = \frac{1}{\lambda}[\exp(\beta w_c) - 1]. \tag{14}$$

At the other limit $\lambda \to 0$, or the medium is only slightly clogged, Eq. 13 can be simplified as

$$\Phi = 1 - \exp(-\beta w_c). \tag{15}$$

Notably, Eq. 15 is identical to the empirical equation Eq. 1 proposed by Notebaert et al. (1975) and Tiller et al. (1981). The correlation equation usually employed for medium clogging is thereby a special case for the more general expression of Eq. 13. By the same analogy, the empirical parameter j in Eq. 1 is equivalent to $\beta/w_{c,\mathrm{ref}}$, whose physical implication is that the probability of an open pore C becoming clogged after unit dry mass of cake has been deposited on the unit filter area. The empirical parameter j must lie between zero and unity.

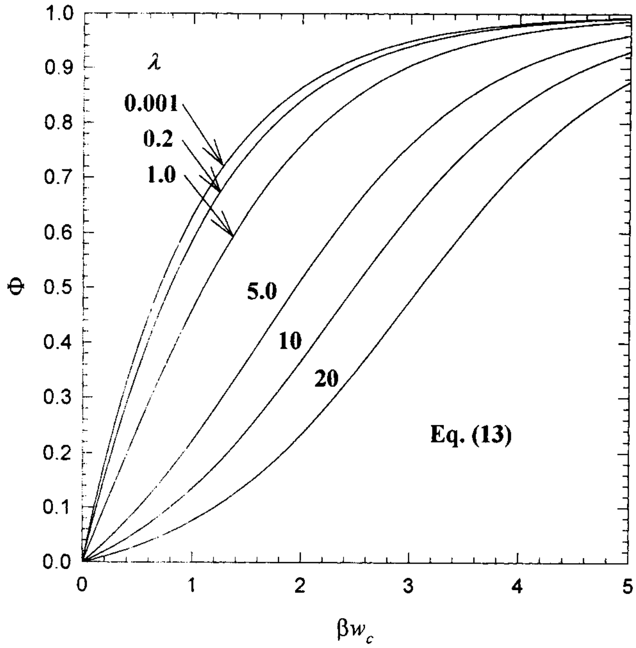

Figure 1. Φ calculated based on Eq. 13 under various λ values.

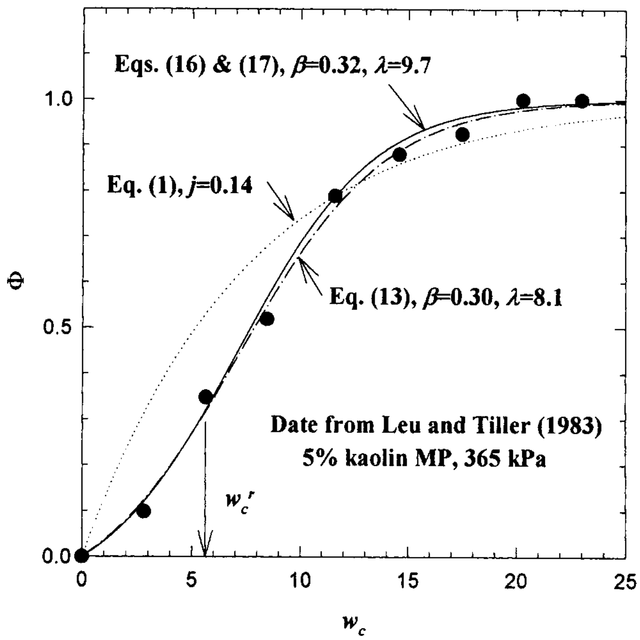

Figure 2. Φ vs. w_c.

Closed data are extracted from Leu and Tiller (1983). 5% Kaolin MP, 365 kPa. Solid curve: Eq. 13 with parameters determined based on Eqs. 16 and 17; dot-dashed curve: Eq. 13 with parameters determined by nonlinear regression analysis; dotted curve: Eq. 1.

Comparison with Experiment

Recorded instances of well-controlled experimental data are rare in the open literature. In this work, the two sets of experimental data drawn from Figures 5 and 7 in Leu and Tiller (1983) verify the present theoretical results. The data points are replotted as solid symbols in Figures 2 and 3. For the former, which is the experimental result for 5% Artisan kaolin MP filtered under 365 kPa, an inflection point is clearly identifiable (Figure 2). For the latter, which is for the same slurry but under 665 kPa, the inflection point is unclear (Figure 3).

The inflection point on the $R_m - w_c$ curve occurs at (w_c^r is the dimensionless dry cake mass per unit area of medium at inflection point)

$$w_c^r = \frac{\ln \lambda}{\beta}. \tag{16}$$

Therefore, the existence of an inflection point indicates that $\lambda > 1$, or the superficial velocity through (clean) pores C is higher than that through pores F. Meanwhile the slope at $w_c = w_c^r$ is

$$\left[\frac{d\Phi}{dw_c}\right]_{w_c = w_c^r} = \frac{\beta(1+\lambda)}{4\lambda}. \tag{17}$$

When an inflection point appears on the $R_m - w_c$ plot, the parameters β and λ can then be estimated based on the corresponding w_c^r value and the slope of the inflection point, and Eqs. 16 and 17. If no inflection point can be identified, nonlinear regression analysis is required to determine these parameters.

The inflection point for data in Figure 2 locates at $w_c^r = 7.1$. The corresponding first derivative is estimated as $d\Phi/dw_c = 0.088$. The λ and β values based on Eqs. 16 and 17 are thereby 9.7 and 0.32, respectively. Figure 2 represents Eq. 13 with these λ and β values as a solid curve. The experimental

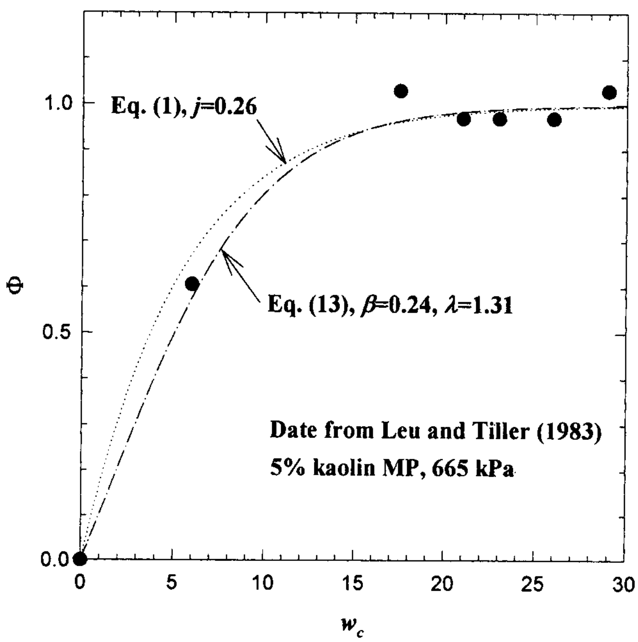

Figure 3. Φ vs. w_c.

Closed data are from Leu and Tiller (1983). 5% Kaolin MP, 665 kPa. Dot-dashed curve: Eq. 13 with parameters determined by nonlinear regression; dotted curve: Eq. 1.

correlation is close. Such a result suggests that up to 90% of pores on the filter medium had become clogged as time neared infinity, while there is an approximately 33% probability of an originally open pore C clogging during the deposit of 1 kg dry cake on 1 m^2 of filter medium.

Nonlinear regression analysis yields the most apposite parameter set as $\lambda = 8.14$ and $\beta = 0.30$, which is close to that based on the inflection point information. Figure 2 also represents Eq. 13 with these parameters as a dot-dashed curve.

The best-fitted j value based on Eq. 1 is 0.14, which is only 44% of the β value as estimated above. The plot for Eq. 1 is indicated by the dot curve in Figure 2. The match between experimental data is not satisfactory.

For the second data set from Leu and Tiller (1983), nonlinear regression analysis is conducted and the best-fitted λ and β values are respectively 1.3 and 0.24. Notably, the probability of an open pore C clogging is similar to that under 365 kPa (0.30), however, the clogging prospect appears far less serious, as evidenced by the much lower λ value. Figure 3 represents Eq. 13 with these parameters as a solid curve. The experimental correlation is also close.

Analysis based on Eq. 1 gives the most apposite j value as 0.262, which is now very close to the β value as determined above (0.24). Figure 3 plots Eq. 1 as a dotted curve. It is no surprise to find that both Eqs. 1 and 13 properly reflect the experimental data, since when λ is small Eq. 13 reduces to Eq. 1.

The pressure effects on medium resistance had been claimed as the cake structure, and would be more resistant to fine particle migration as pressure increases, thereby the medium has less chance to be clogged. This is consistent with the slight decrease in β value, as indicated in Figures 2 and 3. However, the present analysis also demonstrates that the increase in filtration pressure largely reduces the λ value, which is to say that the proportion of open pores that eventually clog decreases with an increase in filtration pressure. This diminution is probably due to the rapid initial clogging of the medium by fine particles under higher pressure, an explanation offered by Leu and Tiller (1983). However, it is also explicable by the other mechanisms, such as the formation of particle bridging above some open pores thus preventing further clogging.

In summary, the newly proposed medium clogging equation (Eq. 13) is superior to the empirical correlation (Eq. 1) for two main reasons. First, the theoretical background of Eq. 13 is sound and the physical implications for all the parameters involved are evidenced. Second, compared with Eq. 1, Eq. 13 can properly interpret experimental data under much wider conditions, and supply more detailed information about the medium clogging.

Conclusions

A two-parameter medium clogging equation (Eq. 13) is proposed and compared with experimental data from Leu and Tiller (1983). We found that our newly proposed equation is superior to the usually employed correlation, Eq. 1, since for the former (a) the theoretical background is sound and the physical interpretations for all the parameters involved are clear, and (b) the applicable range is wider. However, if clogging is not serious, the empirical correlation can continue to provide good data matching and functions well in estimating the medium clogging factor.

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